PHISICS OF URBANISM: THE FRACTAL GROWTH AND DISTRIBUTION OF THE ROMANIAN CITIES AND TOWNS

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Abstract. *In some previous works (M. Gligor and L. Gligor, 2004, 2008) we considered the fractal distribution of cities in Romania by population and area of the urban perimeter. The dataset was taken according to the 2002 census, referring to 265 urban settlements. Subsequently, there were officially declared an additional 55 towns (Wikipedia.org). Today (September, 2011) in Romania there are 320 towns. In the present paper, we demonstrate that using the updated dataset, the basic features of distributions remain essentially the same. In the second part of the paper, the Central Places Theory, the diffusion-limited aggregation and the self-organized criticality mechanisms are investigated by means of some numerical simulations and the last two are found to fit better the urban perimeter growth.*

Keywords: Zipf law, master equation, diffusion-limited aggregation, self-organization.

1. HISTORICAL FRAMEWORK

More than seven decades ago, Englewood Cliffs from New Jersey published a pioneering book by Christaller (1933), where several key questions were for the first time posed: “What type of dynamics describe the growing of the urban locations?” and, further: “Are there laws which determine the number, size and distribution of towns?” The Christaller’s theory – the so-called *central places theory (CPT)*, later developed by Beckman (1968) among others, describes the urban morphology in the terms of the Euclidean geometry considering that the urban development is structured around a *central business district.* From this he deduced that settlements would tend to form in a triangular/hexagonal lattice, this being the most efficient pattern for travel between settlements.

In fact, Christaller’s theory is an important brick in a larger theoretical edifice, namely *the location theory*. Before him, Alfred Weber (1909) formulated a least cost theory of industrial location which tried to explain and predict the location pattern of the industry at a macro-scale. It emphasized that firms seek a site of minimum transport and labor cost, by taking into account several economic factors as the point of optimal transportation based on the costs of distance to the “material index”, the labor distortion, the agglomeration and de-agglomeration.

A complementary approach of the CPT was formulated by August Lösch (1940). While Christaller was starting, in effect, with the largest market area and then turned to commodities with ever smaller market areas, Lösch considered first the commodity with the smallest market area and then introduced other commodities with successively larger market areas. Thus while Christaller’s approach is an inductive one, Lösch’s model is essentially a deductive one. In Lösch’s theory the deviations from optimal spatial layouts for individual commodities are relatively small, and the more flexible distribution of functions between centers permits smaller centers to provide goods and services to larger centers (that is not allowed in Christaller’s approach).

Essentially, in order to avoid the inconstancies, both Christaller and Lösch theories had to make some basic simplifying assumptions such as:

a) The framework of the model consists of an isotropic (all flat), homogeneous, unbounded limitless surface (abstract space), an evenly distributed population, and evenly distributed resources;

b) All consumers have a similar purchasing power and demand for goods and services;

c) There is only one type of transport and this would be equally easy in all directions; transport cost is proportional to distance traveled;

d) There are no external economies or diseconomies permitted, in shopping or in production which could distort the systems of hexagons;

e) No statements about the sizes of central places are possible (except, in the case of Christaller, that each higher order central place is at least as large as all lower order central places);

f) There can be no Thünen-type ring formation because of the need to have an even distribution of demand;

g) Industrial and service production cannot consume any space; otherwise factor prices for land would be different in different-size centers.

All the above assumptions can be hardly considered realistic, especially for the urban systems in the developing countries. Here the configuration is strongly influenced by local factors, such as climate, topography, history of development, technological improvement and personal preference of consumers and suppliers. Economic status of consumers in an area is also crucial in a developing country. Consumers of higher economic status tend to be more mobile and therefore bypass centers providing only lower order goods. On the other hand, the purchasing power and density affect the spacing of centers and hierarchical arrangements.

Obviously, the applicability of the CPT is drastically limited by two factors the exclusive using of Euclidean varieties as lines and surfaces. The modeling of the urban perimeter in Euclidean terms leads to results in strong discrepancy with the empirical evidence, both for large cities and for small towns configurations. That is why some modern approaches consider that the random cluster models and the fractal properties derived from them constitute necessary ingredients in modeling urban development.

The roots of these approaches are placed in the second half of the XX century, when B. Mandelbrot (1975) opened the door towards a more realistic description of many natural and social phenomena by introducing the mathematical varieties with fractional dimensions, usually called *fractals*. In the fractal theories, the dimension has a higher degree of generality than in Euclidean it has. For instance, considering an object M which can be decomposed in N parts, each one in the ratio r (r<1) with the whole object, the (self-similarity) dimension is defined as:

$$D = \frac{\log N}{\log(1/r)} \tag{1}$$

Thus, the dimension associated to the object M may appear as a fractional number.

It is well-known that the distinctive feature of the fractal object is *self-similarity i.e.* it displays the same aspect regardless the scale at which it is viewed. This geometrical property is mathematically expressed in the *scaling* of the characteristic functions $F(x)$ describing the fractal object: $F(cx) = c^{\alpha}F(x)$. The parameter α completely describes the object geometric features; this is like saying that a simple number is able to characterize a very complicated shape. Moreover, the fractal character of the system may be reflected not only in its geometric properties, but also in its characteristic distribution functions. In this way, numerous natural systems display a fractal structure: the mountain ranges, river networks, coastlines, etc. The word "fractal" here means that some correlation functions show non-trivial power law behavior.

In their classical paper on *self-organized criticality (S.O.C.)*, Bak, Tang and Wiesenfield (1987) argued that the dynamics which give rise to the robust power-law correlations seen in the non-equilibrium steady states in nature must not involve any fine-tuning of parameters. It must be such that the systems under their natural evolution are driven to a state at the boundary between the stable and unstable states. Such a state then shows long-range temporal-temporal fluctuations similar to those in equilibrium critical phenomena. Bak et al. (1987) proposed a simple example of a system whose natural dynamics drives it towards, and then maintains it, at the edge of stability: a sand pile. When the average slope of the sand pile is larger than a certain value θ_c, addition of a small amount of sand often results in an avalanche whose size is of the order of the system size, while in a pile where the average slope is θ_c, the response to addition of sand is less predictable. In the steady state of this process the sand may be added to the system at a constant small rate, but it leaves the system in a very irregular manner, with long periods of apparent inactivity interspersed by events which may vary in size and which occur at unpredictable intervals. A long power law tail characterizes the typical distribution of the frequency and size of the avalanches, with an eventual cut-off determined by the system size.

The "random cluster" models (Batty & Xie, 1996) consider the cities growth as the growth of two-dimensional aggregates of particles – problem of particular interest in physics of disordered media. In particular, the model of *diffusion-limited aggregation (DLA)* (Witten Jr. & Sander, 1981) has been applied to describe urban growth, and results in tree-like dendritic structures, which have a core or "central business district". The DLA model predicts that there exists only one large fractal cluster that is screened from incoming "development units" (people, capital, resources, etc).

The DLA model predicts that the urban population density decreases from the centre to the periphery as a power law:

$$\rho \sim r^{D-2} \tag{2}$$

where r is the radial distance from the core and D is the fractal dimension of the model.

Alternatively, an exponential decay is considered in the so-called *correlated percolation model (CPM)* (Makse, Havlin & Stanley, 1995; 1998), where the spatial correlations in urban settlements are also embodied. A modified version of the DLA model in which the cluster density decays as a complementary error function was recently elaborated (Pica Ciamarra & Coniglio, 2006).

Clearly, the CPM offers the best description of the great urban agglomerations (e.g. London, Berlin). Nonetheless, it is a geographical stylized fact that the small villages, at least the ones situated in a plane environment, are roughly concentrically structured, so they seem to be better fitted by the DLA model. It might be of interest to study what happens when the cities are formed by merging some independent developed villages. Bucharest, the analyzed city in the present paper, is one example.

On the other hand, the fractal behavior may be found not in the spatial structure itself, but is manifested through the power-law dependence between some physical observables. Examples include the earthquakes, the fluid turbulence, etc. Note that a lot of time-series from the social sciences, *e.g.* stock market price variations both for the greatest stock markets (Mantegna & Stanley, 2000) and for some emerging ones (Gligor, 2004) display power-law tails in their power spectra (the so-called "1/f noise").

In this new meaning of fractality, let us recall that almost six decades ago, an important result for the urbanism was pointed out: the population and area distributions of cities and towns follows power law behavior (Zipf, 1949). Indeed, if n_s is the number of cities having the population s, then:

$$R(s) = \int_s^{\infty} n_x dx \tag{3}$$

defines the *rank* of the city in a hierarchy: the largest city has R = 1; the second largest R = 2, etc. Zipf found that R is a function of s, which can be inverted as:

$$s(R) \sim R^{-\gamma} \tag{4}$$

with $\gamma \approx 1$.

The first striking property of the above result (known today as "the Zipf's law for cities" is the scale invariance, reflecting an underlying fractal structure; the second consists in universality: the statistical datasets shows that the law is valid for many different societies and during various time periods.

The universality of the power law behavior suggests the possibility of study the urban system by tools that do not depend in an explicit way on the concrete nature of the interactions between its elementary constituents. Gabaix (1999) suggested that the behavior described by Eq. (4) can be explained by assuming an auto-catalytic process characterized by the rule that the growth of each individual entity is proportional to its present size. Nonetheless, few years later, Blank and Solomon (2000) showed that a growing system with a fixed number of components and a fixed smallest component size cannot converge to a power law. Instead, by fixing the minimal population to a certain fraction of the average, they defined the so called "generalized Lotka-Volterra process" with variable number of components, which converges to a power law for a very wide range of parameters. Moreover, in a very large subset of this range, they obtained for the power law exponent the special value 1 specific for the cities population distribution.

The power law distribution is dramatically cut at the upper end, i.e. in the region of the small size towns. Taking into account this aspect, Malacarne, Mendes and Lenzi (2002) suggested the q-exponential distribution (derived from the generalized nonextensive statistical mechanics) as a possible alternative to the power law. However, the cut off can be simply due to some marginal effects related to the finiteness of sampling. At present, the Zipf's law is generally accepted as an empirical fact describing quite different societies (Mulianta, Situngkir, & Surya, 2004; Newman, 2005, and references therein; Moura Jr. & Ribeiro, 2006).

In the following section we show that the Zipf's law can be easily derived by supposing that the development process is Markovian. It is well-known that Markovian stochastic processes can be described by a master equation. In the last decades Weidlich and Haag have successfully introduced this formalism for the description of social processes (Weidlich & Haag, 1983; Weidlich, 1991) like opinion formation (Haag, 1989), migration (Haag & Weidlich, 1984), agglomeration (Weidlich & Haag, 1987) and settlement processes (Weidlich, 1997), and have shown how well-known outcomes might well arise in a dynamic context. Particularly important are the evolution of the mean value and quasi-mean value equations that can depict expected outcomes over time (Weidlich, 2002). These equations operate in a stochastic framework, which is supposed to represent the actions of the individuals or other lower-level units in the system. Following synergetics, trends in order parameters usually determine the overall outcomes.

Some inherent difficulties with the above approaches should be noted. One that has been hinted at is the relative lack of empirical work related to or based on socio-dynamics. In general it is not very easy to empirically estimate the many of the transition probabilities that are crucial to many of the models. On the other hand, the above approaches seem to leave out the possibility that the trends of the order parameters themselves may be altered by changes in individual behavior. In any case, all models have their limits, and socio-dynamic approaches only deal with aggregated phenomena rather than individual outcomes (Weidlich, 2002).

The main points of our research are outlined below. In Section 2 we argue that the Zipf's distributions of cities populations and areas can be derived from the basic assumption that the development process is Markovian, without other additional constraints. This result, well known for the largest cities in all over the world, is empirically tested using data referring to Romania as an example of developing country. Section 3 brings into discussion the shape of the urban perimeter by means of some numerical simulations of DLA and SOC models, versus the empirical founded structure of the largest urban settlement in Romania, namely the capital Bucharest. We find that the basic assumptions of CPT simply do not work. No hexagonal structure can be found in the central places disposition. Instead, the real structure is found to be well fitted by DLA and SOC simulations, and may be entirely explained by particular historical and economic facts of evolution. The last section summarizes the findings and draws some conclusions.

2. THE POPULATION AND AREA DISTRIBUTIONS OF CITIES AND TOWNS

2.1 The stochastic model

Let us consider N towns, and let be s_i the size of the i-th city (expressed as number of citizens as well as units of urban area). The model is built using the general framework of the master equation. We assign the transition rates for the growth $\Psi_+(s_i)$ or decrease $\Psi_-(s_i)$ of the size s_i. In other words, $\Psi_+(s_i)$ is the probability that a new citizen arrives (or a new economic/residential unit-area location is created) in the city i in the time interval $(t, t + dt)$, so that $s_i \to s_i + 1$. Analogous, $\Psi_-(s_i)$ is the probability that one of the s citizens departs (or a unit-area location is left) in the same time interval, so that $s_i \to s_i - 1$.

We introduce now the average number $n(s, t)$ of cities of size s at time t, for a *given N*. The quantity $n(s, t)$ satisfies the master equation:

$$\frac{\partial n(s,t)}{\partial t} = \psi_-(s+1)n(s+1,t) - \psi_-(s)n(s,t) + \psi_+(s-1)n(s-1,t) - \psi_+(s)n(s,t) \quad (5)$$

where $\partial n(s, t)/\partial t$ is the variation of n. The parameters of the model are the transition rates $\Psi_\pm(s_i)$.

If the total number of cities N is considered not be constant, at least an additional parameter must be introduced (e.g. Marsili & Zhang, 1998; Blank and Solomon, 2000), describing the probability that a citizen leaves the system (or a unit-area location is created outside of the system). However, a new city formation implies an *allocation* process, strongly depending on external conditions which cannot be simply included in this approach (but might be an interesting challenge for further works). Moreover, the time scale of this process certainly exceeds the time scale of the statistical data recording.

Thus, as in the study of the most interacting-agent systems, we are firstly interested in finding the stationary solution of the master equation, for which *s* and *N* are constant on average and:

$$\frac{\partial n(s,t)}{\partial t} = 0 \tag{6}$$

In this case $n(s, t) \equiv n(s)$, *i.e.* the quantities *n* and Ψ do not depend explicitly on the time. Equation (5) becomes:

$$\psi_-(s+1)n(s+1) - \psi_-(s)n(s) + \psi_+(s-1)n(s-1) - \psi_+(s)n(s) = 0 \tag{7}$$

The simplest way to take into account the interactions among agents is assuming these interactions pair-wise type, so that: $\Psi \sim s^2$. This assumption simply means that all the *s* city units are in interaction each other, displaying a fully connected social network. In the simplest way, choosing $\Psi_-(s) = k_1 \cdot s^2$ and respectively $\Psi_+(s) = k_2 \cdot s^2$, a straightforward calculus leads to:

$$n(s) = C / s^2 \tag{2}$$

where $C = N \Big/ \sum_{i=1}^{N} s_i^{-2}$. Using the rank relation (3) one finds: $R(s) \sim s^{-1}$, or, inverting: $s(R) \sim 1/R$, that is the usual form of the Zipf's law.

2.2 The empirical dataset

The empirical data referring to the urban area and population of the Romanian cities and towns were supplied by the most recent census performed by Romanian Institute of Statistics (2003). A number of 320 large and medium size towns are ordered by decreasing the urban area, and by decreasing the urban population.

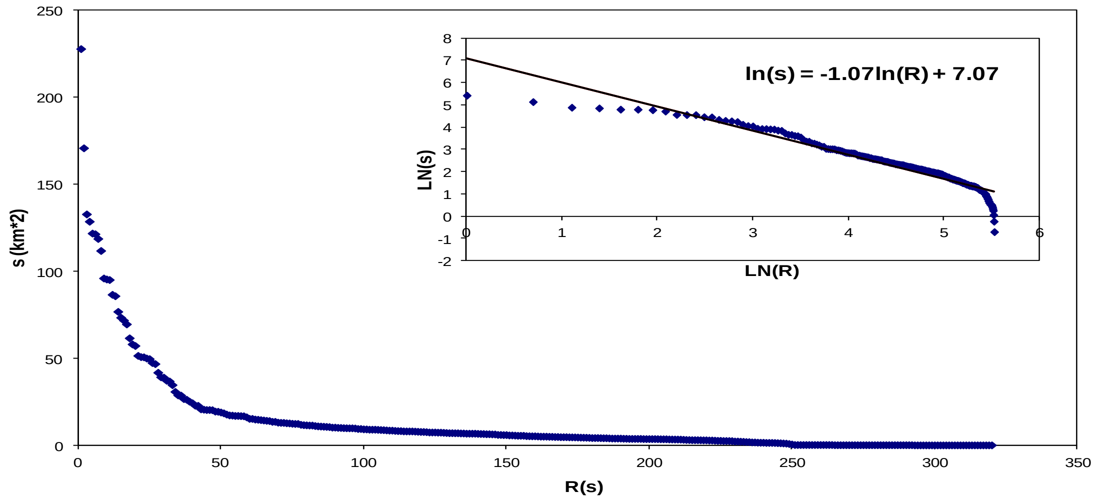

Fig. 1 The urban area distribution for 320 Romanian cities and towns. Inset: the Zipf plot.

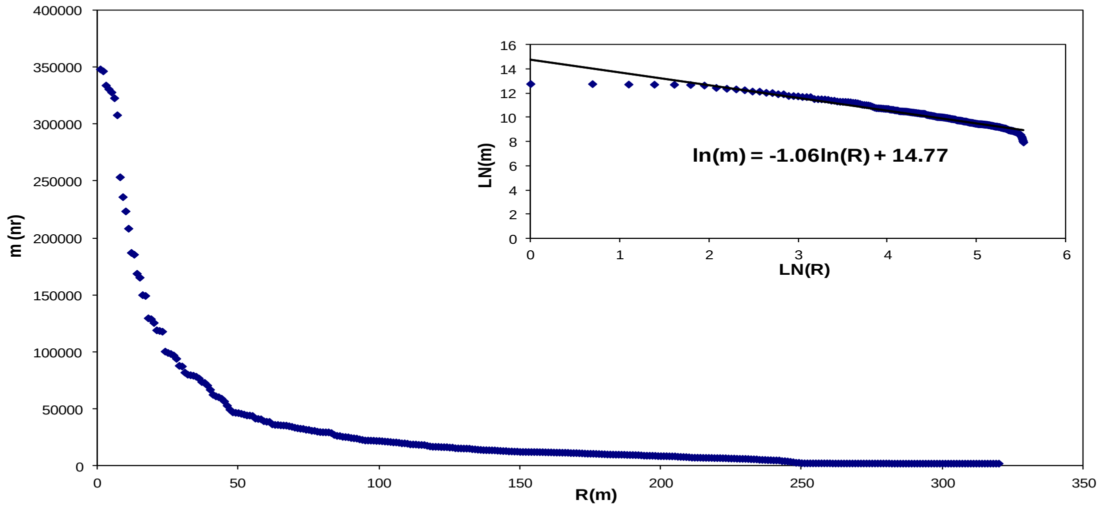

Fig. 2 The urban population distribution for 320 Romanian cities and towns. Inset: the Zipf plot.

The data are well fitted by power laws, leading to the scaling exponents: μ = 1,07 (for the urban area – Fig. 1) and γ = 1,06 (for the urban population – Fig. 2). These results are in well agreement with the similar ones reported in the literature cited in Introduction.

One can see a good agreement of fitting as regards to the Pearson product moment correlation coefficient through the given data points $(R^2)_1$ = 0.92 for the area distribution, and $(R^2)_2$ = 0.94 for the population distribution. The standard errors are σ_1 = 1.07 and σ_2 = 1.05 respectively. According with the Chebyshev's theorem, removing the points situated at more than $\pm 4\sigma$ from the fitted curve, i.e. the “outliers” (9 points in Fig. 1 and 7 points in Fig. 2), one gets $(R^2)_1$ = 0.98 and $(R^2)_2$ = 0.99 respectively. The χ^2 statistic test applied to the remainder points gives more than 98% confidence interval in both cases.

Taking into account the above results we can conclude that the both distributions are well fitted by the Zipf’s law. The fact that in any sampling a threshold minimal value is a priori chosen imposes a biased consideration of the values situated *around* the threshold and can explain the distribution cut-off.

3. THE URBAN PERIMETER MODELING

3.1 Numerical simulations of the growth process

The simulations are performed model on a lattice, which we take for simplicity to be the two dimensional square lattice. The system evolves in discrete time. In the first version (Fig. 3), the simulation follows the slightly modified DLA model: we start from a central site containing a large number of particles. Now we introduce a new particle at a large distance from the seed, and let it perform a random walk. Ultimately, that second particle will either escape to infinity or contact the seed, to which it will stick irreversibly.

Now introduce a third particle into the system and allow it to walk randomly until it either sticks to the two-particle cluster or escapes to infinity. In addition, at each time step, some particles are moved from the central place to the neighboring sites with a power-law decay probability.

In the second version (Fig. 4), one starts from the sand pile model with a uniform distribution of heights (Bak et al. 1987). There is a positive integer variable at each site of the lattice, called the height of the sand pile at that site. At each time step a site is picked randomly, and its height z_i is increased by unity. If the site height is larger than a critical value z_c, the site relaxes by toppling whereby z_c grains leave the site, and each of the four neighboring sites gets $z_c/4$ grains. In case of toppling at a site at the boundary of the lattice, grains falling “outside” the lattice are *not* removed from the system, but they are added randomly to the highest ones. This process continues until all sites are stable.The spatial distribution of “avalanches”/”relaxation” processes are followed at two different times of simulation

3.2 Comparison with the real data

As shown, in the DLA model, only a large central place or large cluster is generated. The cluster generated by this process are both highly branched and fractal (Fig. 3). The cluster's fractal structure arises because the faster growing parts of the cluster shield the other parts, which therefore become less accessible to incoming particles. A new arriving random particle is far more likely to attach to one of the tips of the cluster shown in figure 1a than to penetrate deeply into one of the cluster’s "fjords" without first contacting any surface site.

However, a real urban area is rather composed of central places that are spatially distributed following a certain hierarchy, thus the sand pile model of evolution (Fig. 4a and 4b) offer a more realistic description of the real urban perimeter shown in Fig. 5.

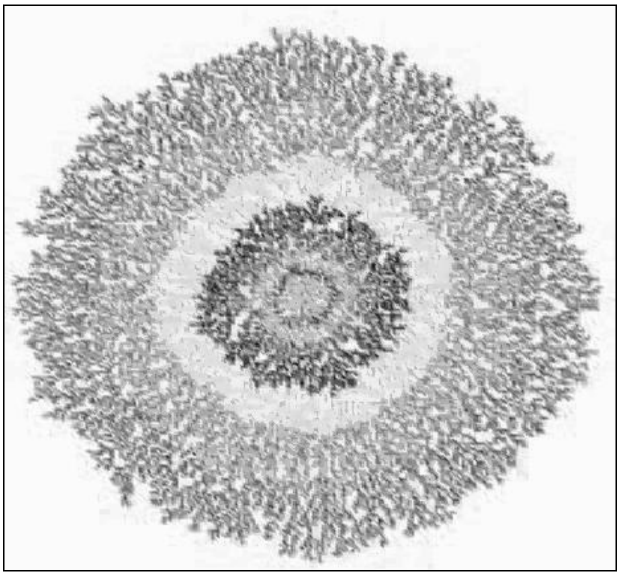

Fig. 3 A numerical simulation of a growth process in a dendritic-like structure, from the DLA model (Eq. 2 with *D* = 1.7). The growth begins in the centre and extends to the periphery.

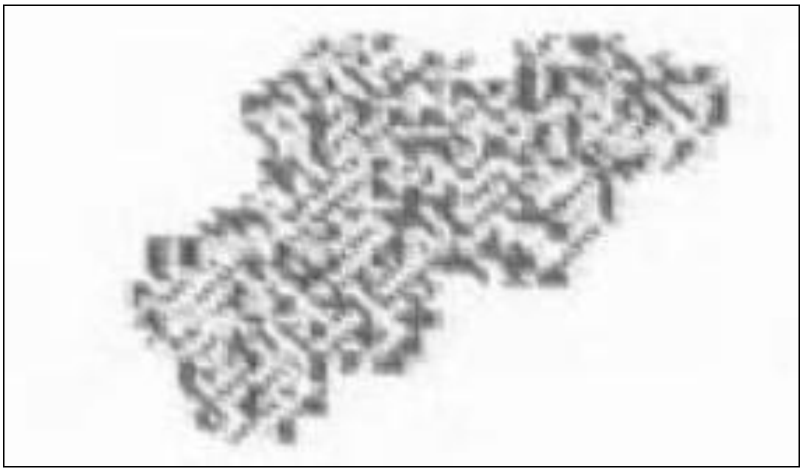

Fig. 4a

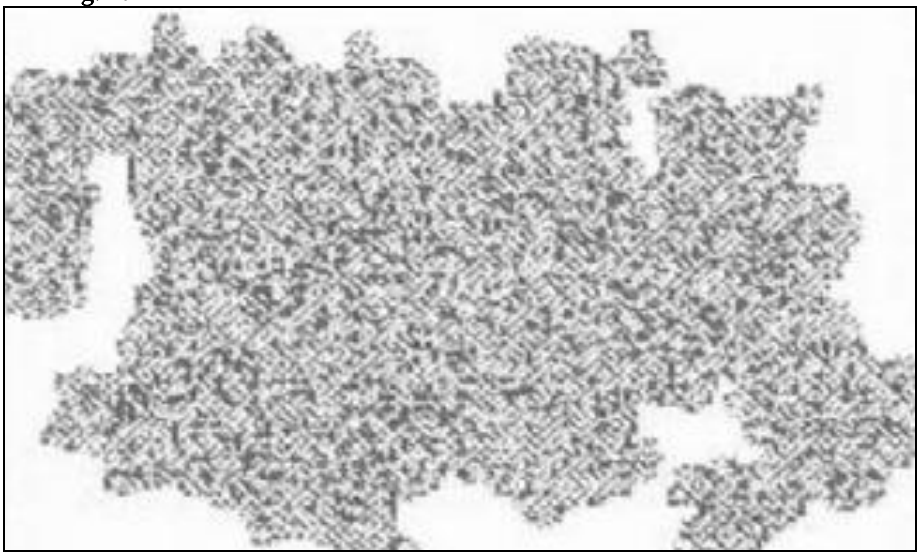

Fig. 4b

Fig. 4 The result of the numerical simulation of a growth process in the sand pile model after **(a)** $n = 10^2$; **(b)** $n = 10^3$ simulation time steps.

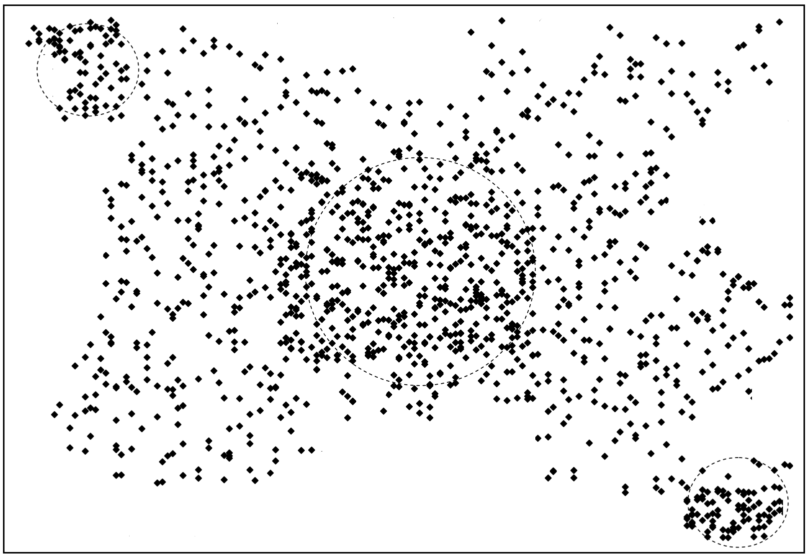

Fig. 5 The structure of Bucharest in 1935.

The residential and economic unit coordinates are obtained by dividing the map in 250 × 250 screen squares. Data was supplied by Museum of Bucharest City (Bucharest) and Machedon and Scoffham (1999).

One can also see in Fig. 5 that the basic assumptions of CPT simply do not work. No hexagonal structure can be found in the central places disposition. Instead, we found one first order and two second order central places, in a quasi linear disposition. This structure can be exclusively explained by historical facts, taking into account that the city was formed by fusion of three independent old villages that must be seen as independent centers of growth.

Each independent center has developed a dendritic-like structure around itself, as predicted by DLA model. Moreover, the density of commercial units decreases following a power law dependence on the distance from the local center.

On the other hand, the actual shape of the urban perimeter is well described by a SOC process, namely the spatial extending of the avalanches processes in the sand pile model. This result indicates that the first steps of any urban development are governed by “trial and error” principle rather than economic efficiency reasons. The randomness of the avalanches locations means here that the firms’ growth or failure result from the interaction of numerous social, political, economic factors, too many to be considered separately as explanatory facts, as well as from the agents idiosyncratic behavior.

4. CONCLUSIONS

In the present paper some basic ideas of the fractal city theory have been briefly reviewed. Particularly the questions of urban perimeter growth and towns’ distribution were pointed out. While an increasing amount of literature is devoted to the large cities structure and distribution, a relatively low interest has been so far given in the study of medium and small-size urban locations, especially those situated in the developing countries. Generally here are not mega-polis-like cities and the most urban centers are formed by merging some small units (villages).

In this case, we found that:

(i) The Zipf’s law can be directly derived from the assumption of the Markovian process of development without other auxiliary hypothesis.

(ii) The power law distribution with exponent roughly unit was found to be valid for the urban settlements distribution in a developing country as well as it was found in the previous studies referring to the cities distribution of the developed countries;

(iii) The basic assumptions of the Central Places Theory are not fulfilled in the case of cities formed by merging old village-like settlements. Particularly, the regular hexagonal structure predicted by CPT cannot be found.

(iv) The Diffusion-Limited Aggregation model fits very the growth process around the local (second-order) poles.

(v) The self-organized sand pile model seems to fit very well the urban perimeter shape. This fact can bring into discussion the relative importance of various economic, political and geographical particular factors, as well as the intrinsic competition between order and hazard.

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